# Extension of Topology Optimization Method Based on Regularized Level Set Function to Solving 3-D Electromagnetic Field System

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**A topology optimization (TO) method is an effective tool for the practical design of electrical machines. TO was conventionally achieved by using the material density. However, material-density-based TO may produce problematic solutions; e.g., a large number of gray scale areas, and unfeasible shapes. On the other hand, when the TO is based on the advection of level set functions, it is superior in terms of both the continuity and practicability of the optimized solution. This paper proposes a level-set-based TO method for solving three-dimensional (3-D) magnetic field problems with magnetic nonlinearity.**

***Index Terms*—Finite element method, level set function, regularization, topology optimization.**

## I. Introduction

The topology optimization (TO) approach is one of most effective tools for the conceptual design of electrical machines. Material-density-based TO was proposed by Bendsøe, primarily in the field of structural analysis [1]. The method was subsequently applied to the electromagnetic field optimization problem [2]-[4]. Although material-density-based TO has the advantage of offering fast convergence, the shape of the converged solution may occasionally be unfeasible.

This difficulty was overcome by basing the TO on the advection of level set functions as proposed by Sethian [5]. In this method, the interface between the material body and free space is determined by the zero-isosurface of the level set function. Because the occurrence of gray scale elements is restricted in the neighborhood of the zero-isosurface, the shape to which the optimization finally converges is more feasible than that derived from material-density-based TO. However, there is a drawback to the handling of the advection term in Hamilton-Jacobi equation arises in the update of the level set function. To enhance the numerical stability of the Hamilton-Jacobi equation, Park introduced an artificial diffusion term [6]. As a result, the Hamilton-Jacobi equation became similar to the reaction-diffusion equation arising in the phase field method [7]. Although the resultant topology is reasonable and practical, the drawback of this approach is that the coefficient of the diffusion term additionally requires tuning.

Then, Yamasaki proposed a regularization technique to eliminate the advection term from the Hamilton-Jacobi equation [8], and Min successfully applied this technique to the two-dimensional (2-D) magnetic field problem [9]. However, the effect of the regularization technique on the electromagnetic field problem is not clearly shown in [9]. Therefore, the validity of the regularization technique of the level set function is clearly revealed in this paper, and the TO based on this regularized level set function is newly extended to the 3-D nonlinear magnetic field problem.

## II. 3-D Optimization Scheme Based on Regularization

### A. Hamilton-Jacobi Equation

The zero-isosurface of level set function of position vector $\boldsymbol{x}$ at time $t$ is formulated as follows:

$$\phi(\boldsymbol{x},t)=0\,. \tag{1}$$

Then, the partial derivative of (1) with respect to $t$ is

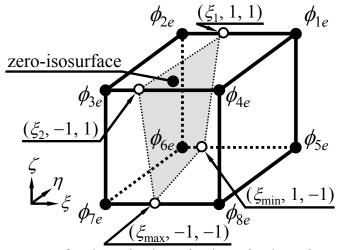
$$\frac{\partial\phi(\boldsymbol{x},t)}{\partial t}+|\nabla\phi|\,v_n=0 \qquad \left(\because\ v_n=\boldsymbol{n}\cdot\frac{\partial\boldsymbol{x}}{\partial t}\right), \tag{2}$$

where $\boldsymbol{n}$ is the normal vector of unit length, and $v_n$ is the normal component of the velocity vector.

### B. Regularization of Level Set Function

Because the characteristics of the level set function are based on the signed distance, the absolute value of the advection term in (2) can be set to 1.0. However, using the design sensitivity when the level set function is updated would cause the signed distance property to collapse during the optimization process.

Thus, a regularization to retain the signed distance property is achieved by extracting the zero-isosurface of the level set function from every optimization step. Fig. 1 shows an example of an extracted zero-isosurface, which is shaded and composed of four local coordinates ($\xi_1$, 1, 1), ($\xi_2$, −1, 1), ($\xi_{\max}$, −1, −1), and ($\xi_{\min}$, 1, −1). The sequence of points on the zero-isosurface is generated by extracting this isosurface element-by-element. Next, the value of the level set function of all




cond.: $\phi_{1e}>0$, $\phi_{2e}<0$, $\phi_{3e}<0$, $\phi_{4e}>0$, $\phi_{5e}>0$, $\phi_{6e}<0$, $\phi_{7e}<0$, $\phi_{8e}>0$

Fig. 1. Three-dimensional regularization of level set function.

nodes in the design domain was redetermined by using the zero-isosurface in every optimization step.

Applying its regularization, the absolute value of advection term $|\nabla\phi|$ in (2) can be transformed to 1.0 completely. Then, (2) can be easily solved by the finite element method (FEM) supported by a forward Euler method.

## III. 3-D Optimization Problem

Fig. 2 shows the 3-D actuator model, in which the optimization goal is to maximize the magnetic energy stored in the armature under the condition that the iron core volume $V(\phi)$ is less than $V_0$. The optimization problem is formulated:

$$\begin{aligned}&\min.\ f(\phi)=\left(\frac{1}{2}\int_{\Omega_t}\boldsymbol{B}^T\nu\boldsymbol{B}\,dV\right)^{-1}\\&\text{s.t.}\quad g(\phi)=V(\phi)-V_0\le 0\quad\left(\because\ V(\phi)=\int_{\Omega_d}H(\phi)\,dV\right)\end{aligned},\quad(3)$$

where $\boldsymbol{B}$ is the magnetic flux density, $\nu$ is the reluctivity, $\Omega_t$ is the target area, $\Omega_d$ is the design domain, and $H(\phi)$ is the Heaviside function, respectively. In the design domain, the nonlinear property of EU67 is considered to the iron core in design domain. The finite element mesh with nonconforming connection is specified as follows: hexahedral 1st order edge element with 583,779 elements, 606,624 nodes, 606,624 DoF of FEM, and 352,000 DoF of TO, is adopted.

When the constraint condition is taken into account by using a Lagrange multiplier, the convergence characteristic of the objective function is frequently oscillated. If $g(\phi)$ is greater than zero, the constant bias $\gamma$ added to all $\phi$ in the design domain is evaluated by solving the nonlinear equation:

$$g(\phi+\gamma)=0,\quad(4)$$

which can be solved by using the Newton method with respect to $\gamma$. This equation is used to perform the volume correction if $g(\phi)$ is greater than zero during the optimization process.

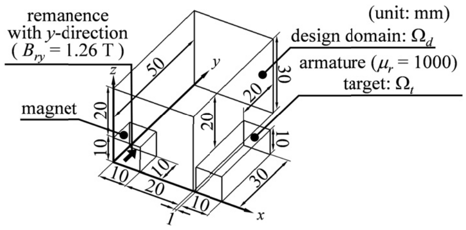



Fig. 2. Three-dimensional actuator model.

## IV. Results and Discussion

Three optimization cases were prepared as shown in TABLE I. The maximum number of TO iterations was uniformly set to 500 for each of these cases, and the resulting 3-D topologies are shown in Fig. 3. In each of the three cases, the initial topology was set to assume a spherical shape as shown in Fig. 3 (a). When regularization of the level set function was not applied to the TO, topology growth did not proceed beyond a certain point as shown in Fig. 3 (b), because the smooth distribution of the level set function is suppressed considerably. On the other hand, application of the regularization had the effect of drastically improving both the converged topologies as shown in Fig. 3 (c). Further, when the magnetic property was linear, the converged value $f(\phi)^{-1}$ was also improved as shown in TABLE I. With magnetic nonlinearity, the cross-section of the optimized topology exceeded that of the linear case as shown in Fig. 3 (d). Finally, the practical application will be described in a four-page paper.

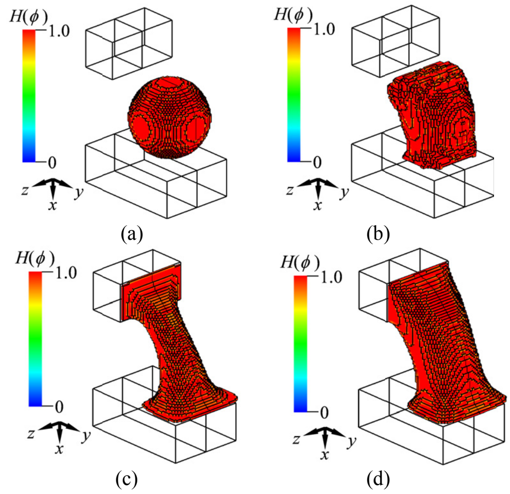



Fig. 3. Distributions of the Heaviside function: (a) initial topology, (b) optimization result without regularization (magnetic property: linear), (c) optimization result with regularization (magnetic property: linear), and (d) optimization result with regularization (magnetic property: nonlinear).

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TABLE I Optimization Results

| dim. | magnetic nonlinearity | regularization | max. it. | $f(\phi^{(\text{max.it.})})^{-1}$ [mJ] | $V(\phi^{(\text{max.it.})})/V_0$ | gray scale ratio [%] | elapsed time [h] |
|---|---|---|---|---|---|---|---|
| 3-D | – | – | 500 | 0.110 | 0.80 | 0.34 | 11.9 |
| | | ✔ | | 0.605 | 0.47 | 0.63 | 12.6 |
| | ✔ | | | 0.535 | 1.00 | 0.89 | 27.9 |

CPU: Intel Core i7-2600K 4.2 GHz with overclocking & 16.0 GB